1. Assume $d_1[n] = e^{j \frac{2\pi}{8} n}$ and $d_3[n] = e^{j \frac{2\pi}{8} 3n}$ be two sinusoids. Then
   
   (a) (5 Points) $<d_1, d_3> = ?$, Note that $\sum_{n=0}^{N-1} a_n = \frac{1-a^N}{1-a}$

   
   $<d_1, d_3> = \sum_{n=0}^{7} d_1[n]d_3^*[n] = \sum_{n=0}^{7} e^{j \frac{2\pi}{8} n} e^{-j \frac{2\pi}{8} 3n} = e^{-j \frac{\pi}{2} n} = \frac{1-e^{-j \frac{\pi}{2} 8}}{1-e^{-j \pi/2}} = 1 + j = 0$

   (b) (20 Points) $||d_1||_2^2 = ?, ||d_3||_2^2 = ?, ||d_1||_\infty = ?, ||d_3||_\infty = ?$

   
   $||d_1||_2 = <d_1, d_1^{1/2} = e^{j \frac{2\pi}{8} n}, e^{-j \frac{2\pi}{8} n} >^{1/2} = \left( \sum_{n=0}^{7} 1 \right)^{1/2} = \sqrt{8}$

   $||d_3||_2 = <d_3, d_3^{1/2} = e^{j \frac{2\pi}{8} 3n}, e^{-j \frac{2\pi}{8} 3n} >^{1/2} = \left( \sum_{n=0}^{7} 1 \right)^{1/2} = \sqrt{8}$

   $||d_3||_\infty = ||d_1||_\infty = 1$

   (c) (5 Points) $\left\| d_1 \frac{1}{\sqrt{8}} \right\|_2 = ?$

   
   $\left\| \frac{1}{\sqrt{8}} d_1 \right\|_2 = \frac{1}{\sqrt{8}} \sqrt{8} = 1$

2. (20 Points) Prove that the offset system $y[n] = x[2n]$ with an input $x$, and output, $y$ is linear

   When $x[n]$ is applied we get $y[n] = x[2n]$. Therefore let us apply $\hat{x}[n] = \alpha x[n]$. Then we obtain $\hat{y}[n] = \hat{x}[2n] = \alpha x[2n] = \alpha y[n]$. Hence, we proved that the scaling property holds. Next we need to show that additivity also holds. Assume $x_1[n] \implies y_1[n] = x_1[2n]$ and $x_2[n] \implies y_2[n] = x_2[2n]$. Then applying $\hat{x}[n] = x_1[n] + x_2[n]$, leads to $\hat{y}[n] = \hat{x}[2n] = x_1[2n] + x_2[2n] = y_1[n] + y_2[n]$. Therefore the system is LINEAR.
3. (20 Points) The impulse response $h$ of an DT LTI system is as shown in figure. If a signal $x$ which is given below is applied to this system, what shall be the output signal $y$. What is the moral of the story? As we can observe from figures $x[n]$ and $y[n]$ that, this system works as an edge detector and find outs where the major changes occur in the input signal $x[n]$. 
4. (30 Points) The impulse responses of DT LTI systems \( H_1 \) and \( H_2 \) are shown below. Calculate the output \( y \) if the finite-length signal \( x \) is applied to the interconnection given in figure. **Hint:** Do not use convolution. First, obtain system matrices \( H_1 \) and \( H_2 \). Then think about the cascade connection.

We can easily construct system matrices \( H_1 \) and \( H_2 \) as follows:

\[
H_1 = \begin{bmatrix}
0 & 3 & 2 & 1 \\
1 & 0 & 3 & 2 \\
2 & 1 & 0 & 3 \\
3 & 2 & 1 & 0
\end{bmatrix}, \quad H_2 = \begin{bmatrix}
1 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1
\end{bmatrix}
\]

Then \( y = H_2(H_1x) = H_2H_1x = [8 \ 8 \ 4 \ 4]^T \)